

**Constructive Theory of Orthogonality and Applications
(CTOA 2018)**

Short conference dedicated to the 70th Anniversary of Professor
Gradimir V. Milovanović

Tuesday, May 29th
University of Belgrade, Faculty of Mechanical Engineering,
Department of Mathematics

PROGRAM

10:00-12:00 Conference room: 211 (2nd floor)

10:00 *Rada M. Mutavdžić*

Error bounds for Kronrod extension of generalizations of Micchelli-Rivlin quadrature formula for analytic functions

10:30 *Dušan Lj. Đukić*

Truncated generalized averaged Gaussian quadratures and their internality

11:00 *Jelena D. Tomanović*

Error Estimates for Certain Cubature Rules

11:30 *Tatjana V. Tomović*

An optimal set of quadrature rules for trigonometric polynomials in the sense of Borges

12:00-13:00 Conference room: 211 (2nd floor)

12:00 *Miodrag M. Spalević*

Gradimir Milovanović Expert of Numerical Analysis and Scientific Computing

12:15 *Gradimir V. Milovanović*

Constructive Theory of Orthogonality and Applications

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I

Abstracts

Constructive Theory of Orthogonality and Applications

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May 29, 2018

Constructive theory of orthogonal polynomials was developed in eighties in a series of papers by Walter Gautschi. It opened the door for extensive computational work on orthogonal polynomials and their applications not only in mathematics, but in other computational and applied sciences. Beside the basic procedures for numerical generation of coefficients in the three-term recurrence relation for orthogonal polynomials for arbitrary measures, in this lecture we present some details on the stability analysis of such algorithms, Christoffel modifications of the measure and corresponding algorithms, as well as available software. This theory enables the construction of many new classes of strongly non-classical orthogonal polynomials (very often with certain exotic weights), development of other types of orthogonality (s and σ -orthogonality, orthogonality on radial rays, Sobolev type of orthogonality, multiple orthogonality, etc.), applications in diverse areas of applied and numerical analysis (numerical integration, interpolation, integral equations, ...), approximation theory (moment-preserving spline approximation, ...), integration of fast oscillating functions, summation of slowly convergent series, etc. Particular attention will be paid to some of these issues.

Error bounds for Kronrod extension of generalizations of Micchelli-Rivlin quadrature formula for analytic functions

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May 29, 2018

We consider Kronrod extension of generalizations of the Micchelli-Rivlin quadrature formula, with the highest algebraic degree of precision, for the Fourier-Chebyshev coefficients. For analytic functions the remainder term of these quadrature formulas can be represented as a contour integral with a complex kernel. We study the kernel, on elliptic contours with foci at the points ∓ 1 and a sum of semi-axes $\rho > 1$, for the mentioned quadrature formulas. We derive the L^∞ -error bounds and the L^1 -error bounds for these quadrature formulas. Finally, bounds resulting from expanding the remainder term in these quadrature formulas are obtained. Numerical examples which illustrate the calculation of these error bounds are included.

This is joint work with Aleksandar V. Pejčev and Miodrag M. Spalević.

Truncated generalized averaged Gaussian quadratures and their internality

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Generalized averaged Gaussian quadrature formulas, introduced by Spalević, may yield a smaller error than Gauss quadrature rules. When moments or modified moments are difficult to compute, these formulas can serve as good substitutes. However, generalized averaged Gaussian quadrature formulas may have external nodes, i.e. nodes outside the convex hull of the measure corresponding to the Gauss rules. This would make them unusable when the domain of the integrand is limited to this convex hull. In this paper we investigate whether removing some of the last rows and columns of the matrices determining generalized averaged Gaussian quadrature rules will produce quadrature rules with no external nodes.

The results that will be presented are a joint work with Lothar Reichel (Department of Mathematical Sciences, Kent State University) and Miodrag Spalević (University of Belgrade, Faculty of Mechanical Engineering).

Error Estimates for Certain Cubature Rules

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We estimate the error of selected cubature formulae constructed by the product of Gauss quadrature rules. The cases of multiple and (hyper-)surface integrals over n -dimensional cube, simplex, sphere and ball are considered. The error estimates are obtained as the absolute value of the difference between cubature formula constructed by the product of Gauss quadrature rules and cubature formula constructed by the product of corresponding Gauss-Kronrod or corresponding generalized averaged Gaussian quadrature rules. Generalized averaged Gaussian quadrature rule \hat{G}_{2l+1} is $(2l+1)$ -point quadrature formula. It has $2l+1$ nodes and the nodes of the corresponding Gauss rule G_l with l nodes form a subset, similar to the situation for the $(2l+1)$ -point Gauss-Kronrod rule H_{2l+1} associated with G_l . The advantages of \hat{G}_{2l+1} are that it exists also when H_{2l+1} does not, and that the numerical construction of \hat{G}_{2l+1} , based on recently proposed effective numerical procedure, is simpler than the construction of H_{2l+1} .

This is joint work of Miodrag Spalević, Davorka Jandrić and Jelena Tomanović.

An optimal set of quadrature rules for trigonometric polynomials in the sense of Borges

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In this talk we consider an optimal set of quadrature rules with an odd number of nodes for trigonometric polynomials in the sense of Borges [Numer. Math. **67** (1994), 271–288]. We introduce multiple orthogonal trigonometric polynomials of semi-integer degree in order to characterize an optimal set of quadrature rules. The main properties of such a kind of multiple orthogonal trigonometric polynomials are given. Theoretical results are illustrated by some numerical examples.

This is joint work with Gradimir V. Milovanović and Marija P. Stanić.